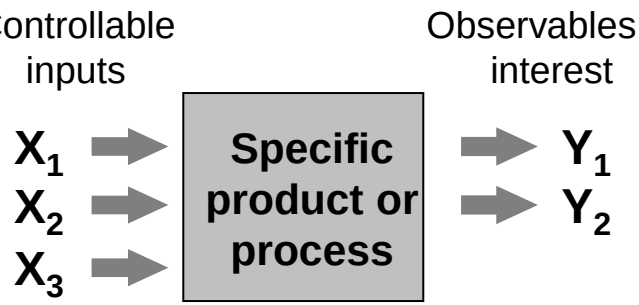
Design of Experiments (DOE)

Problem
How to determine the factors controlling an output?

Difficulty
Work with an SME

- **Design of Experiments** (DOE) is a cost effective statistical approach that quantifies the effect of inputs on outputs.
- DOE makes specific changes to inputs and observes the resulting outputs.

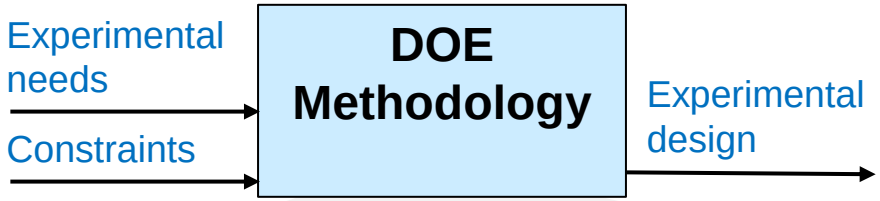
- **This is the system**



- **Problem** – find inputs to minimize (say) the observables, using as few tests as possible.
- **Solution** – use the model below – observables depend on the inputs, with (usually) the earlier terms being more important than later terms:

$$Y_i = \overline{Y} + \sum_i a_i X_i + \sum_{i,j} b_{ij} X_i X_j + \ldots$$

overall mean | main effects | Two-way interactions



1. Define the test objective(s)
   - What is the overall problem?
2. Select and quantify the critical response(s)
   - What observables are of concern?
3. Design the experiment (incorporate features such as randomization, replication, and blocking)
   - Define all the inputs for each test
4. Perform all the tests and collect the data
5. Analyze the data (use a SW package)
6. Interpret the results
7. Verify the predicted outcome

Terminology
- Each input has "**levels**" (perhaps 3 different values for X_1 and 4 different values for X_2).
- A "**full factorial design**" has a test for every possible combination of levels.
- A "**partial or fractional factorial design**" uses a subset of the tests in the full factorial design.

DOE – Example – Golf score

Example from: Jack B. ReVelle, *Manufacturing Handbook of Best Practices: An Innovation, Productivity, and Quality Focus*, https://books.google.com/books?id=_EfMBQAAQBAJ&pg=PP5

(1) Want to minimize a golf score based on the following 7 controllable inputs. (Note that each input has 2 levels.)

	Inputs	Level (-1)	Level (+1)
A	Age of clubs	Old	New
B	Time of day	AM	PM
C	Use golf cart	No	Yes
D	Practice at driving range	Yes	No
E	Drink during game	Yes	No
F	Type of ball	Wilson	Titleist
G	Use of caddy	Yes	No

(2) A full factorial seven factor design at 2 levels has 2^7=128 experiments. Instead, use 8 experiments:

		Inputs (encoded)							Observation			
		A	B	C	D	E	F	G	Y_1	Y_2	Y_3	Y_4
Experiment	#1	-1	-1	-1	-1	-1	-1	-1				
	#2	-1	-1	1	-1	1	1	1				
	#3	1	1	-1	1	-1	1	1				
	#4	-1	1	1	1	1	-1	-1				
	#5	1	-1	-1	1	1	-1	1				
	#6	1	-1	1	1	-1	1	-1				
	#7	1	1	-1	-1	1	1	-1				
	#8	1	1	1	-1	-1	-1	1				

Observed values go here

(3) Convert to input levels and perform the experiments. Potential observable values are shown.

		Input values							Golf score
		A	B	C	D	E	F	G	Y_1
Experiment	#1	Old	AM	No	Yes	Yes	Wilson	Yes	84
	#2	Old	AM	Yes	Yes	No	Titleist	No	96
	#3	Old	PM	No	No	Yes	Titleist	No	89
	#4	Old	PM	Yes	No	No	Wilson	Yes	97
	#5	New	AM	No	No	No	Wilson	No	94
	#6	New	AM	Yes	No	Yes	Titleist	Yes	91
	#7	New	PM	No	Yes	No	Titleist	Yes	94
	#8	New	PM	Yes	Yes	Yes	Wilson	No	92

(4) A simple analysis finds the most important inputs (to leading order) – these cause the largest change.

	Inputs	Levels	Totals	Means	Effect
A	Age of clubs	Old	366	91.50	1.25
		New	371	92.75	
B	Time of day	AM	365	91.25	1.75
		PM	372	93.00	
C	Use golf cart	No	361	90.25	3.75
		Yes	376	94.00	
D	Practice at driving range	Yes	366	91.50	1.25
		No	371	92.75	
E	Drink during game	Yes	356	89.00	6.25
		No	381	95.25	
F	Type of ball	Wilson	367	91.75	0.75
		Titleist	370	92.50	
G	Use of caddy	Yes	366	91.50	1.25
		No	371	92.75	

DOE – Notes

Slide 1

1. The goal of DOE is to obtain useful results using a minimal number of tests.
2. Inputs are also called “factors”.
3. Use SMEs and SW tools to find a test plan.
4. The tests should be designed to capture the expected terms that are important in the input/output relationship – which may be the linear terms (the first term after the constant) or the higher order terms (the later terms)
5. The "one factor at a time" approach is inefficient compared to changing multiple factor levels simultaneously.
6. Usually, an initial “screening design” is used to reduce a long list of potentially important factors and interactions to only a few important effects.
7. Selecting or creating an appropriate partial or fractional factorial design
    A. depends on how many terms in the model are anticipated to be important
    B. depends on many other factors, such as possible relationships between the inputs
    C. is something of an art!

Slide 2

1. In this example the golf score is to be minimized.
2. The chosen design has only 8 tests; this is far fewer than the 128 in the full factorial design. For this example, that few tests may be adequate.
    - The resulting predictions, that drinking and golf cart usage are the most important factors, should be verified
3. Note that, for the tests performed, each level for each input appears exactly four times.