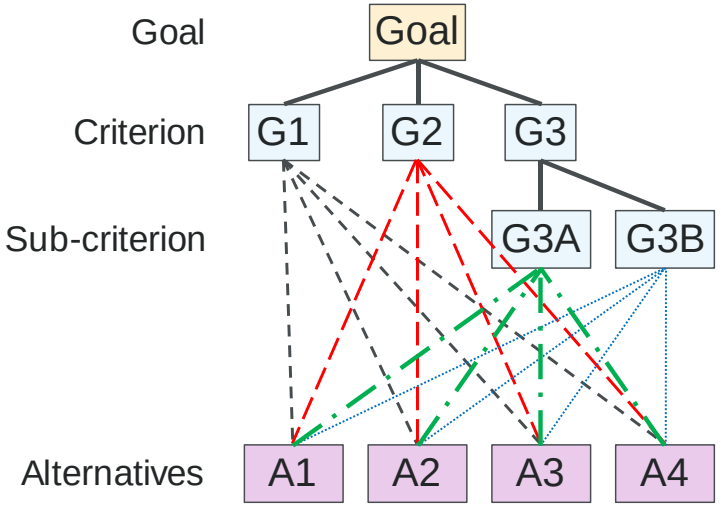
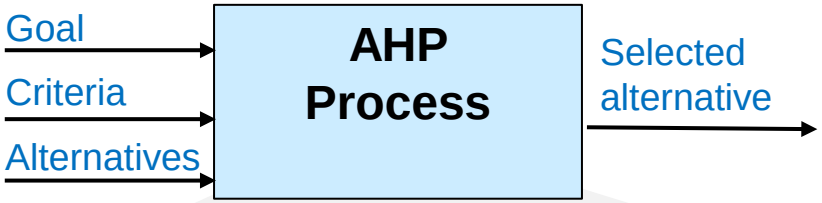
Analytical Hierarchy Process (AHP)

Problem
How to choose among multiple alternatives?

Difficulty
Work with an SME

- The **Analytic Hierarchy Process** (AHP) is a method for making decisions under multiple and complex criteria.
- AHP is easy to use since stakeholders only need to perform pairwise comparisons, assigning values 1-9.
- The pairwise comparisons are performed between all the criteria, between each set of sub-criteria, and between all the alternatives.





1. Define the goal.
2. Define the criteria (simple or hierarchical)
3. Define the alternatives.
4. Determine the weights amongst the criteria, sub-criteria, and alternatives (for each criteria) using pairwise comparison.
5. Use SW to convert pairwise comparisons into weights and confirm consistency.
6. Use SW to combine priorities and obtain overall weights for the alternatives.

Pairwise Comparison Scale	
Intensity	**Definition**
1	Equal Importance
3	Moderate Importance
5	Strong importance
7	Very strong importance
9	Extreme importance

Sample scale with corresponding text

AHP – Example – Selecting a Leader

- **Choosing a leader from among 3 candidates.**
- **Use 3 criteria: experience, education, and teaching ability.**

(1) Compare the selection criteria pairwise to determine their priorities. (If "A" is preferred over "B" by a factor of N, then "B" is preferred over "A" by a factor of 1/N)

Obtain matrix of pairwise results → these are AHP inputs

Criteria	Experience	Education	Teaching	Weights
Experience	1	5	9	0.751
Education	1/5	1	7	0.178
Teaching	1/9	1/7	1	0.070
		inconsistency		1.5%

From the 3-by-3 matrix, AHP finds the weights (0.75, 0.18, 0.07); which sum to 1.

The value "1/9" means the stakeholders think "Teaching is 1/9th as important as Experience" which is the same as "Experience is 9 times more important than Teaching."

(2) For each of the 3 criteria compare the candidates pairwise, to create a 3-by-3 matrix. Then, AHP determines the weights (blue boxes) for each criteria.

Experience	Alex	Beth	Chris	Weights
Alex	1	3	1/5	0.188
Beth	1/3	1	1/7	0.081
Chris	5	7	1	0.731
		inconsistency		3.2%

Education	Alex	Beth	Chris	Weights
Alex	1	1/5	5	0.207
Beth	5	1	9	0.735
Chris	1/5	1/9	1	0.058
		inconsistency		5.9%

Teaching	Alex	Beth	Chris	Weights
Alex	1	1/3	5	0.265
Beth	3	1	9	0.672
Chris	1/5	1/9	1	0.063
		inconsistency		1.5%

(3) Weight the alternative priorities, for each of the criteria, by that criteria's AHP weights. (See the red computation.)

Candidate	Experience	Education	Teaching	Row sum
Alex	0.141	0.037	0.019	**0.197**
Beth	0.061	0.131	0.047	**0.239**
Chris	0.549	0.010	0.004	**0.563**

Largest row sum value: Chris is the best choice

0.061 = 0.751*0.081
0.549 = 0.751*0.731

(for values, see red boxes in steps (1) and (2))

AHP – Notes

Slide 1

1. AHP was developed by Thomas L. Saaty.
2. AHP is easier to show than to describe.
3. Any range of values can be used for Intensity, not just {1,3,5,7,9}.
4. A data inconsistency occurs, for example, when the pairwise comparisons indicate that “A” is preferred to “B”, and “B” is preferred to “C”, yet “C” is preferred to “A”.
5. AHP software determines an “inconsistency;” if this value is larger than 10%, then the pairwise comparisons should be reviewed.
6. Like probabilities, weights are numbers between zero and one, without units.
7. AHP can address hierarchical criteria. For example, when buying a truck, the carrying capacity and the number of seats may be important. The carrying capacity may depend on both the size of the cargo area and the weight it can carry.
8. AHP computations are best left to software packages. (AHP weights are the eigenvector corresponding to the largest eigenvector of the pairwise comparison matrix.)

Slide 2

1. The example has a simple set of criteria, with no hierarchy.
2. There are three computational steps:
   A. Determine the criteria weights (by specifying pairwise comparisons)
   B. Determine weights of the alternatives for each of criteria (by specifying pairwise comparisons)
   C. Combine the above results.
3. In this example, each of the inconsistencies is determined to be less than 10%. Hence, we accept the comparisons, and the resulting weights, as being consistent.
4. The best option has the largest overall value. If two options have similar large values, then other techniques might be used to decide between those two options.

Recommended web sites for more information

- https://www.transparentchoice.com/analytic-hierarchy-process
- https://www.pmi.org/learning/library/analytic-hierarchy-process-prioritize-projects-6608