

Analytical Hierarchy Process (AHP)

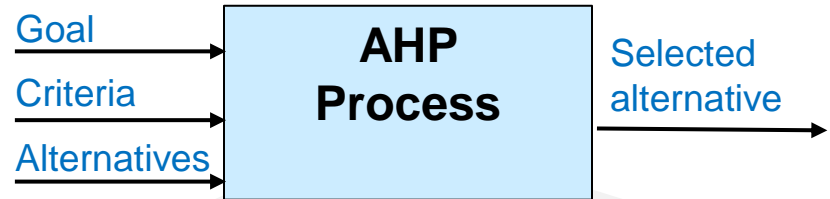
Problem

How to choose among multiple alternatives?

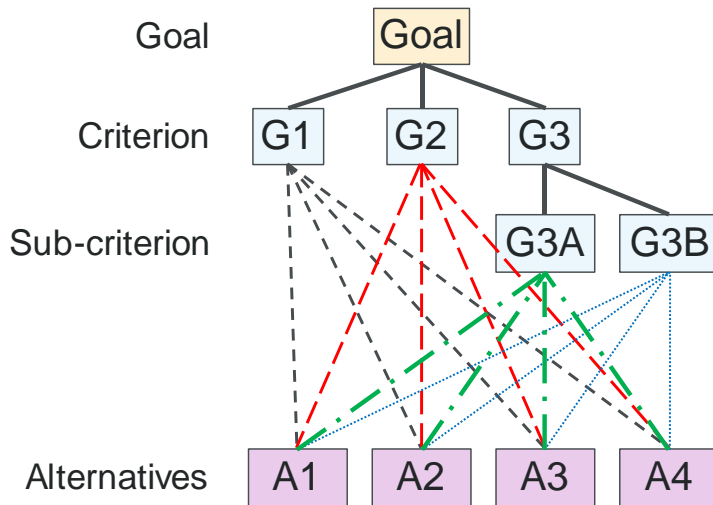
Difficulty

Work with an SME

- The **Analytic Hierarchy Process (AHP)** is a method for making decisions under multiple and complex criteria.
- AHP is easy to use since stakeholders only perform pairwise comparisons, assigning values 1-9.
- The pairwise comparisons are performed between all the criteria, between each set of sub-criteria, and between the alternatives.



1. Define the goal.
2. Define the criteria (simple or hierarchical)
3. Define the alternatives.
4. Determine the priorities amongst the criteria, sub-criteria, and alternatives (for each criteria) using pairwise comparison.
5. Use SW to convert pairwise comparisons into priorities and confirm consistency.
6. Use SW to combine priorities and obtain overall priorities for the alternatives.
7. Use SW to perform a sensitivity analysis.



Pairwise Comparison Scale	
Intensity	Definition
1	Equal Importance
3	Moderate Importance
5	Strong importance
7	Very strong importance
9	Extreme importance

AHP – Example – choose a leader – from Wikipedia

From https://en.wikipedia.org/wiki/Analytic_hierarchy_process_%E2%80%93_leader_example

- Want to choose a leader
- Have 4 criteria: experience, education, charisma, age

(1) Compare the criteria pairwise to determine their priorities. (If “A” is preferred over “B” by a factor of N, then “B” is preferred over “A” by a factor of 1/N)

Team pairwise results – process inputs

Criteria	Experience	Education	Charisma	Age	Priority
Experience	1	4	3	7	0.547
Education	1/4	1	1/3	3	0.127
Charisma	1/3	3	1	5	0.270
Age	1/7	1/3	1/5	1	0.056
	Sum of Priorities				1.000
	Inconsistency				0.044

Priorities found by SW

Always sums to one

Small, good!

(2) The stakeholders compare the alternatives, pairwise, for each criteria.

Experience	Tom	Dick	Harry	Priority
Tom	1	1/4	4	0.217
Dick	4	1	9	0.717
Harry	1/4	1/9	1	0.066
	Sum of Priorities			1.000
	Inconsistency			0.035

Education	Tom	Dick	Harry	Priority
Tom	1	3	1/5	0.188
Dick	1/3	1	1/7	0.081
Harry	5	7	1	0.731
	Sum of Priorities			1.000
	Inconsistency			0.062

Charisma	Tom	Dick	Harry	Priority
Tom	1	5	9	0.743
Dick	1/5	1	4	0.194
Harry	1/9	1/4	1	0.063
	Sum of Priorities			1.000
	Inconsistency			0.069

Age	Tom	Dick	Harry	Priority
Tom	1	1/3	5	0.265
Dick	3	1	9	0.672
Harry	1/5	1/9	1	0.063
	Sum of Priorities			1.000
	Inconsistency			0.028

Team pairwise results – process inputs

(3) Weight the alternative priorities, for each of the criteria, by that criteria's priority.

Candidate	Priority with Respect to				Goal
	Experience	Education	Charisma	Age	
Tom	0.119	0.024	0.201	0.015	0.358
Dick	0.392	0.010	0.052	0.038	0.492
Harry	0.036	0.093	0.017	0.004	0.149
Totals:	0.547	0.127	0.270	0.056	1.000

$$0.119 = 0.547 * 0.217$$

$$0.392 = 0.547 * 0.717$$

sum of each row's values

largest value

Dick is the best choice

AHP – Notes

Slide 1

1. AHP was developed by Thomas L. Saaty in the 1970s.
2. AHP computations involve linear algebra (eigenvalues) and are best left to specialized software packages. (There are many.)
3. Any values 1-9 can be used for Intensity, not just {1,3,5,7,9}.
4. The computations are easier to show than to describe.
5. A data inconsistency occurs when the pairwise comparisons indicate that “A” is preferred to “B”, and “B” is preferred to “C”, yet “C” is preferred to “A”.
6. AHP software usually determines an overall inconsistency; if this value is large than the pairwise comparisons should be reviewed.
7. Like probabilities, priorities are numbers between zero and one, without units.
8. AHP can handle multiple criteria and, using stakeholder input, determine the relative importance of each of the criteria. For example, when buying a truck both the cargo carrying capacity and the number of seats may not be equally important.

Slide 2

1. The example has a simple set of criteria, with no hierarchy.
2. There are three computational steps:
 - A. Determine the criteria priorities
 - B. Determine priorities of the alternatives for each of criteria
 - C. Combine the above results
3. The best option has the largest overall value. If the two options with the largest values were numerically close, then other techniques might be used to decide between them.